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Effect of carbon emission and inflation on inventory model with price dependent demand and time dependent holding cost

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Abstract: This study presents an inventory model with realistic factors such as price-sensitive demand, preservation technology investment, carbon emission, inflation and time-dependent holding cost. Carbon emissions are accounted for during holding and deterioration processes, integrating environmental sustainability into the model. Inflation effect is also considered over planning horizon and the holding cost is treated as a time dependent function. The objective function is to maximize the total profit by jointly optimizing the selling price, replenishment cycle, and preservation technology investment. Under inflationary conditions and sustainability limitations, the model offers managerial insights into striking a balance between economic and environmental performance in inventory systems.

Keywords- non-instantaneous items, carbon emission, preservation technology, inflation.

1. Introduction

Inventory system plays a pivotal role in the efficient operation of supply chains, particularly for perishable products that are subject to deterioration over time. Traditional inventory models often assume constant demand, negligible deterioration, and fixed system parameters, which may not reflect real-world complexities. However, in modern supply chain practices, numerous factors such as fluctuating market demand, product perishability, environmental concerns, inflationary trends, and operational costs—must be simultaneously considered to achieve both economic efficiency and sustainability. One of the critical aspects influencing inventory decisions is the rate at which items deteriorate. Deterioration not only leads to direct loss of

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inventory but also increases operational costs and carbon emissions associated with waste disposal and replacement. Recent advancements in preservation technology (PT) have enabled firms to reduce deterioration rates through targeted investments such as improved packaging, refrigeration, or controlled storage environments. By incorporating a preservation technology cost $\varepsilon\varepsilon$, the deterioration rate can be effectively controlled.

Inventory systems now prioritize environmental sustainability. Energy consumption, storage, and the disposal of damaged goods all contribute to carbon emissions. The model can reflect ecological implications and adhere to increasingly strict environmental rules by include carbon emission costs associated with holding and deterioration.

Additionally, over time, inflation has a major impact on cost structures and the value of money in many economies. Particularly over extended planning horizons, ignoring inflation can result in less-than-ideal pricing and replenishment choices. As a result, the model takes inflation's effects on expenses and income into account. In order to account for real-world fluctuations brought on by shifting storage conditions, energy costs, or insurance premiums over time, holding costs are also regarded as time-dependent. The technology streamlines coordination logistics with zero lead time and instantaneous replenishment, concentrating on optimizing dynamic decision variables under practical economic and environmental limitations.

In order to maximize the total present value of profit under inflationary conditions, time-varying holding costs, and carbon emission considerations, this study attempts to construct a comprehensive inventory model that jointly optimizes the selling price, cycle duration, and investment in preservation technology. Through pricing and technology investment methods, the suggested model gives decision-makers strategic insights on how to balance profitability, sustainability, and product freshness. Now in next section we will discuss review of literature.

2. Literature Review

From simple economic order quantity (EOQ) models to complicated frameworks that take into account real-world complexity including deterioration, pricing tactics, environmental implications, and technological interventions, inventory modelling has changed dramatically over the years. With a focus on five main themes deterioration and preservation technology, price-sensitive demand, time-dependent holding costs, inflation in inventory systems, and carbon emissions and sustainability in supply chains this section examines significant advancements in inventory theory that are pertinent to the current investigation. Deterioration of inventory—referring to spoilage, evaporation, obsolescence, or degradation of products—is a critical issue in industries dealing with perishables such as food, pharmaceuticals, and cosmetics. The seminal work of Ghare and Schrader (1963) introduced the concept of exponential decay in inventory systems, laying the foundation for deteriorating inventory models. Later, Covert and Philip (1973) extended this by considering a two-parameter Weibull distribution for the deterioration rate. Buzacott (1975) pioneered the incorporation of inflation into inventory models, analysing its impact under different pricing and discounting scenarios. Researchers such as Goswami and Chaudhuri (1992) and Balkhi and Benkherouf (2008) developed models with time-varying holding costs, showing that ignoring such variability may lead to suboptimal replenishment policies. Urban and Baker (1988) showed that demand often follows a power function of price, typically decreasing as price increases. Mirzazadeh et al. (2009) extended this by developing a stochastic inventory model under inflation and uncertain demand, emphasizing the importance of considering the time value of money. However, traditional models treated deterioration as an exogenous, uncontrollable factor. A significant advancement came with Hsu et al. (2010), who proposed that the rate of deterioration could be reduced through investment in preservation technology (PT). Inflation affects the present value of future costs and revenues, particularly in long-term inventory planning. Benjaafar et al. (2013) provided foundational insights into the relationship between operational decisions and carbon emissions. Sebatjane (2025) developed an inventory model in which they consider sustainable inventory models with different carbon policies.

3. Assumptions and Notations

The following assumptions are considered for this mathematical model-

1. The Replenishment rate is instantaneous.
2. The demand rate is selling price dependent, and it is given as $D(p) = -\frac{\eta}{p^{\beta}}$.
3. The lead time is zero.
4. Carbon emission is considered for holding items and deterioration.
5. Inflation is also considered.
6. Holding cost is considered as time dependent.
7. Reduced deterioration rate considers a function of $\varepsilon, f(\varepsilon) = \theta(1 - e^{-\alpha\varepsilon})$, where ε is the preservation technology cost for reducing deterioration rate in order to preserve the products.
8. $\lambda(\varepsilon) = \big(\theta - f(\varepsilon)\big)$ is the resultant deterioration.

The following notations are used for this work-

Notations	Description
p	Selling price
η	Demand coefficient
β	Demand constant
ε	Preservation technology cost

θ	Deterioration rate
α	Simulation coefficient representing the percentage increase in $f(\varepsilon)$ increase in ε
C_p	Purchasing cost
O	Ordering cost
C_h	Holding cost
C_d	Deterioration cost
S_c	Shortage cost
S_l	Lost sale cost
γ_1	Carbon emission cost due to holding items
γ_2	Carbon emission cost due to deteriorate items
T_1	Time where items start deteriorate
T_2	Time where inventory become zero
T	Total cycle time
$I_1(t)$	Inventory level between time $0 \leq t \leq T_1$
$I_2(t)$	Inventory level between time $T_1 \leq t \leq T_2$
$I_3(t)$	Inventory level between time $T_2 \leq t \leq T$
Decision variable	
p	Selling price
ε	Preservation technology cost
T	Total cycle time

4. Mathematical modelling

At $t = 0$, the inventory level is Q_1 and after that inventory level decrease due to demand till $t = T_1$ and after that inventory level decrease due to demand and deterioration both till $t = T_2$. And then shortage start till $t = T$.

The differential equations setup of this model is-

$$\frac{dI_1(t)}{dt} = -\frac{\eta}{p^\beta}, 0 \leq t \leq T_1$$

$$\frac{dI_2(t)}{dt} = -\frac{\eta}{p^\beta} - \lambda(\varepsilon)I_2(t), T_1 \leq t \leq T_2$$

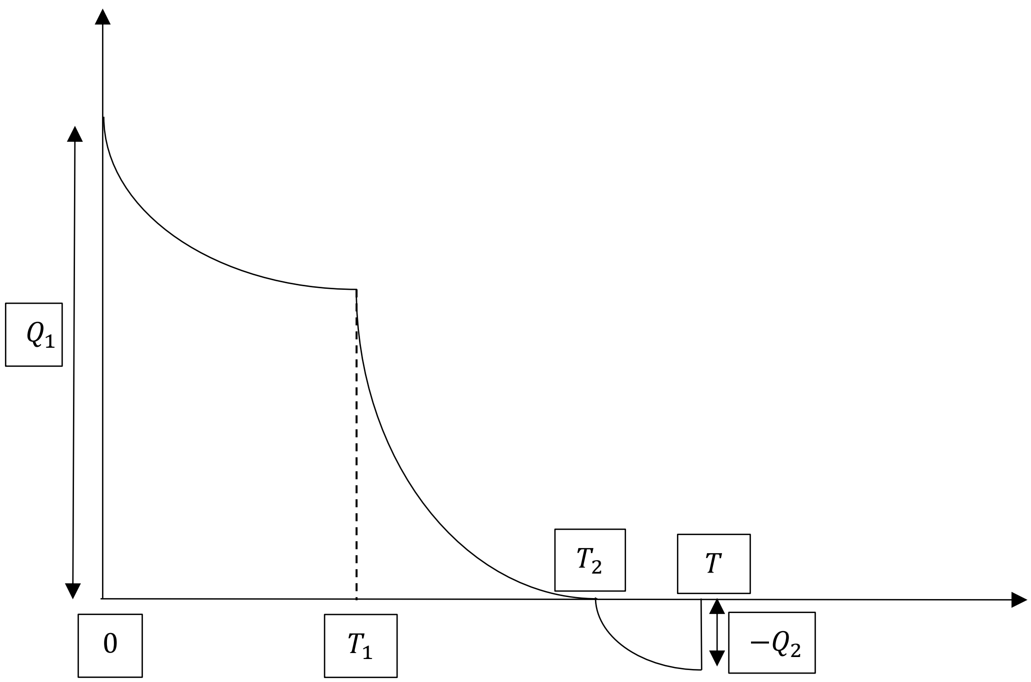

Fig:1- inventory level with respect to time

$$\frac{dI_3(t)}{dt} = -\frac{\eta}{p^{\beta}}, T_2 \le t \le T$$

With boundary conditions-

$$I_1(0) = -Q_1, I_1(T_1) = I_2(T_1), I_2(T_2) = 0, I_3(T) = -Q_2, I_3(T_2) = 0$$

The solutions of these equations with these boundary conditions-

$$I_1(t) = \frac{\eta}{p^{\beta}}(T_1 - t) + \frac{\eta}{p^{\beta}}(e^{\lambda(\varepsilon)(T_2 - T_1)} - 1)$$

$$I_2(t) = \frac{\eta}{p^{\beta}}(e^{\lambda(\varepsilon)(T_2 - t)} - 1)$$

$$I_3(t)=\frac{\eta}{p^\beta}(T_2-t)$$

$$Q_1=\frac{\eta}{p^\beta}T_1+\frac{\eta}{p^\beta}\left(e^{\lambda(\varepsilon)(T_2-T_1)}-1\right)$$

$$Q_2=\frac{\eta}{p^\beta}(T-T_2)$$

Find all inventory cost-

Ordering cost- $OC=O$

Purchasing cost- $PC=C_pQ_1=C_p\left(\frac{\eta}{p^\beta}T_1+\frac{\eta}{p^\beta}\left(e^{\lambda(\varepsilon)(T_2-T_1)}-1\right)\right)$

Holding cost- $HC=C_h\left[\int_0^{T_1}I_1(t)\,te^{-rt}dt+\int_{T_1}^{T_2}I_2(t)te^{-rt}dt+\int_{T_2}^{T}I_3(t)te^{-rt}dt\right]$

$$HC=C_h\left[\frac{\eta}{p^\beta}\left(T_1\left(\frac{T_1e^{-rT_1}}{-r}-\frac{e^{-rT_1}-1}{r^2}\right)-\left(\frac{T_1^2e^{-rT_1}}{-r}-\frac{2T_1e^{-rT_1}}{r^2}-\frac{2\left(e^{-rT_1}-1\right)}{r^3}\right)+\left(e^{\lambda(\varepsilon)(T_2-T_1)}-1\right)\left(\frac{T_1e^{-rT_1}}{-r}+\frac{e^{-rT_1}-1}{r^2}\right)\right)+\frac{\eta}{p^\beta}\left(\frac{T_2e^{-rT_2}-T_1e^{\lambda(\varepsilon)(T_2-T_1)-rT_1}}{-(\lambda(\varepsilon)+r)}-\frac{e^{-rT_2}-e^{\lambda(\varepsilon)(T_2-T_1)-rT_1}}{(\lambda(\varepsilon)+r)^2}+\frac{T_2e^{-rT_2}-T_1e^{-rT_1}}{r}+\frac{e^{-rT_2}-e^{-rT_1}}{r^2}\right)+\frac{\eta}{p^\beta}\left(T_2\left(\frac{Te^{-rT}-T_2e^{-rT_2}}{-r}-\frac{e^{-rT}-e^{-rT_2}}{r^2}\right)-\left(\frac{T^2e^{-rT}-T_2^2e^{-rT_2}}{-r}-\frac{2(Te^{-rT}-T_2e^{-rT_2})}{r^2}-\frac{2(e^{-rT}-e^{-rT_2})}{r^3}\right)\right)\right]$$

Deterioration cost- $DC=C_d\lambda(\varepsilon)\frac{\eta}{p^\beta}\left(\frac{T_2e^{-rT_2}-T_1e^{\lambda(\varepsilon)(T_2-T_1)-rT_1}}{-(\lambda(\varepsilon)+r)}-\frac{e^{-rT_2}-e^{\lambda(\varepsilon)(T_2-T_1)-rT_1}}{(\lambda(\varepsilon)+r)^2}+\frac{T_2e^{-rT_2}-T_1e^{-rT_1}}{r}+\frac{e^{-rT_2}-e^{-rT_1}}{r^2}\right)$

Preservation technology cost- $PTC=\varepsilon$

Shortage cost- $SC=-S_c\int_{T_2}^{T}I_3(t)e^{-rt}dt$

$$SC=-S_c\frac{\eta}{p^\beta}\left(T_2\frac{e^{-rT}-e^{-rT_2}}{-r}-\left(\frac{Te^{-rT}-T_2e^{-rT_2}}{-r}-\frac{e^{-rT}-e^{-rT_2}}{r^2}\right)\right)$$

Lost sale cost- $LSC=C_l\int_{T_2}^{T}(1-\delta)De^{-rt}dt$

$$LSC = C_l(1-\delta)\frac{\eta}{p^{\beta}}\left(\frac{e^{-rT}-e^{-rT_2}}{-r}\right)$$

Carbon emission cost- $CEC = \gamma_1 holding\ items + \gamma_2\ deteriorating\ items$

$$CEC = \gamma_1 \Bigg[\frac{\eta}{p^{\beta}}\Bigg(T_1\left(\frac{T_1e^{-rT_1}}{-r}-\frac{e^{-rT_1}-1}{r^2}\right)-\left(\frac{T_1^2e^{-rT_1}}{-r}-\frac{2T_1e^{-rT_1}}{r^2}-\frac{2\left(e^{-rT_1}-1\right)}{r^3}\right)+\left(e^{\lambda(\varepsilon)(T_2-T_1)}-\right.$$

$$\left.1\right)\left(\frac{T_1e^{-rT_1}}{-r}+\frac{e^{-rT_1}-1}{r^2}\right)\Bigg)+\frac{\eta}{p^{\beta}}\left(\frac{T_2e^{-rT_2}-T_1e^{\lambda(\varepsilon)(T_2-T_1)-rT_1}}{-(\lambda(\varepsilon)+r)}-\frac{e^{-rT_2}-e^{\lambda(\varepsilon)(T_2-T_1)-rT_1}}{(\lambda(\varepsilon)+r)^2}+\frac{T_2e^{-rT_2}-T_1e^{-rT_1}}{r}+\right.$$

$$\left.\frac{e^{-rT_2}-e^{-rT_1}}{r^2}\right)+\frac{\eta}{p^{\beta}}\Bigg(T_2\left(\frac{Te^{-rT}-T_2e^{-rT_2}}{-r}-\frac{e^{-rT}-e^{-rT_2}}{r^2}\right)-\left(\frac{T^2e^{-rT}-T_2^2e^{-rT_2}}{-r}-\frac{2(Te^{-rT}-T_2e^{-rT_2})}{r^2}-\right.$$

$$\left.\frac{2(e^{-rT}-e^{-rT_2})}{r^3}\right)\Bigg)\Bigg]+\gamma_2\left[\lambda(\varepsilon)\frac{\eta}{p^{\beta}}\left(\frac{T_2e^{-rT_2}-T_1e^{\lambda(\varepsilon)(T_2-T_1)-rT_1}}{-(\lambda(\varepsilon)+r)}-\frac{e^{-rT_2}-e^{\lambda(\varepsilon)(T_2-T_1)-rT_1}}{(\lambda(\varepsilon)+r)^2}+\frac{T_2e^{-rT_2}-T_1e^{-rT_1}}{r}+\right.\right.$$

$$\left.\left.\frac{e^{-rT_2}-e^{-rT_1}}{r^2}\right)\right]$$

Total cost-

TC= ordering cost+ purchasing cost+ holding cost+ deterioration cost+ Preservation technology cost+ shortage cost+ lost sale cost + carbon emission cost

$$TC = \frac{1}{T}\Bigg[O + C_p\left(\frac{\eta}{p^{\beta}}T_1+\frac{\eta}{p^{\beta}}\left(e^{\lambda(\varepsilon)(T_2-T_1)}-1\right)\right)+C_h\Bigg[\frac{\eta}{p^{\beta}}\Bigg(T_1\left(\frac{T_1e^{-rT_1}}{-r}-\frac{e^{-rT_1}-1}{r^2}\right)-\left(\frac{T_1^2e^{-rT_1}}{-r}-\right.$$

$$\left.\frac{2T_1e^{-rT_1}}{r^2}-\frac{2\left(e^{-rT_1}-1\right)}{r^3}\right)+\left(e^{\lambda(\varepsilon)(T_2-T_1)}-1\right)\left(\frac{T_1e^{-rT_1}}{-r}+\frac{e^{-rT_1}-1}{r^2}\right)\Bigg)+\frac{\eta}{p^{\beta}}\left(\frac{T_2e^{-rT_2}-T_1e^{\lambda(\varepsilon)(T_2-T_1)-rT_1}}{-(\lambda(\varepsilon)+r)}-\right.$$

$$\left.\frac{e^{-rT_2}-e^{\lambda(\varepsilon)(T_2-T_1)-rT_1}}{(\lambda(\varepsilon)+r)^2}+\frac{T_2e^{-rT_2}-T_1e^{-rT_1}}{r}+\frac{e^{-rT_2}-e^{-rT_1}}{r^2}\right)+\frac{\eta}{p^{\beta}}\Bigg(T_2\left(\frac{Te^{-rT}-T_2e^{-rT_2}}{-r}-\frac{e^{-rT}-e^{-rT_2}}{r^2}\right)-$$

$$\left(\frac{T^2e^{-rT}-T_2^2e^{-rT_2}}{-r}-\frac{2\left(Te^{-rT}-T_2e^{-rT_2}\right)}{r^2}-\frac{2\left(e^{-rT}-e^{-rT_2}\right)}{r^3}\right)\Bigg)\Bigg]+C_d\lambda(\varepsilon)\frac{\eta}{p^{\beta}}\left(\frac{T_2e^{-rT_2}-T_1e^{\lambda(\varepsilon)(T_2-T_1)-rT_1}}{-(\lambda(\varepsilon)+r)}-\right.$$

$$\left.\frac{e^{-rT_2}-e^{\lambda(\varepsilon)(T_2-T_1)-rT_1}}{(\lambda(\varepsilon)+r)^2}+\frac{T_2e^{-rT_2}-T_1e^{-rT_1}}{r}+\frac{e^{-rT_2}-e^{-rT_1}}{r^2}\right)+\varepsilon-S_c\frac{\eta}{p^{\beta}}\Bigg(T_2\frac{e^{-rT}-e^{-rT_2}}{-r}-\left(\frac{Te^{-rT}-T_2e^{-rT_2}}{-r}-\right.$$

$$\left.\frac{e^{-rT}-e^{-rT_2}}{r^2}\right)\Bigg)+C_l(1-\delta)\frac{\eta}{p^{\beta}}\left(\frac{e^{-rT}-e^{-rT_2}}{-r}\right)+\gamma_1\Bigg[\frac{\eta}{p^{\beta}}\Bigg(T_1\left(\frac{T_1e^{-rT_1}}{-r}-\frac{e^{-rT_1}-1}{r^2}\right)-\left(\frac{T_1^2e^{-rT_1}}{-r}-\frac{2T_1e^{-rT_1}}{r^2}-\right.$$

$$\left.\frac{2\left(e^{-rT_1}-1\right)}{r^3}\right)+\left(e^{\lambda(\varepsilon)(T_2-T_1)}-1\right)\left(\frac{T_1e^{-rT_1}}{-r}+\frac{e^{-rT_1}-1}{r^2}\right)\Bigg)+\frac{\eta}{p^{\beta}}\left(\frac{T_2e^{-rT_2}-T_1e^{\lambda(\varepsilon)(T_2-T_1)-rT_1}}{-(\lambda(\varepsilon)+r)}-\right.$$

$$\frac{e^{-rT_2}-e^{\lambda(\varepsilon)(T_2-T_1)-rT_1}}{(\lambda(\varepsilon)+r)^2}+\frac{T_2e^{-rT_2}-T_1e^{-rT_1}}{r}+\frac{e^{-rT_2}-e^{-rT_1}}{r^2}\right)+\frac{\eta}{p^{\beta}}\left(T_2\left(\frac{Te^{-rT}-T_2e^{-rT_2}}{-r}-\frac{e^{-rT}-e^{-rT_2}}{r^2}\right)-$$

$$\left(\frac{T^2e^{-rT}-T_2^2e^{-rT_2}}{-r}-\frac{2\left(Te^{-rT}-T_2e^{-rT_2}\right)}{r^2}-\frac{2\left(e^{-rT}-e^{-rT_2}\right)}{r^3}\right)\right)\Bigg]+\gamma_2\left[\lambda(\varepsilon)\frac{\eta}{p^{\beta}}\left(\frac{T_2e^{-rT_2}-T_1e^{\lambda(\varepsilon)(T_2-T_1)-rT_1}}{-(\lambda(\varepsilon)+r)}-\right.\right.$$

$$\left.\left.\frac{e^{-rT_2}-e^{\lambda(\varepsilon)(T_2-T_1)-rT_1}}{(\lambda(\varepsilon)+r)^2}+\frac{T_2e^{-rT_2}-T_1e^{-rT_1}}{r}+\frac{e^{-rT_2}-e^{-rT_1}}{r^2}\right)\right]\Bigg]$$

Now, in next section we introduce solution methodology that we use for optimal solution of this work.

5. Numerical and graphical analysis

we show the numerical data and graphical representation of this model.

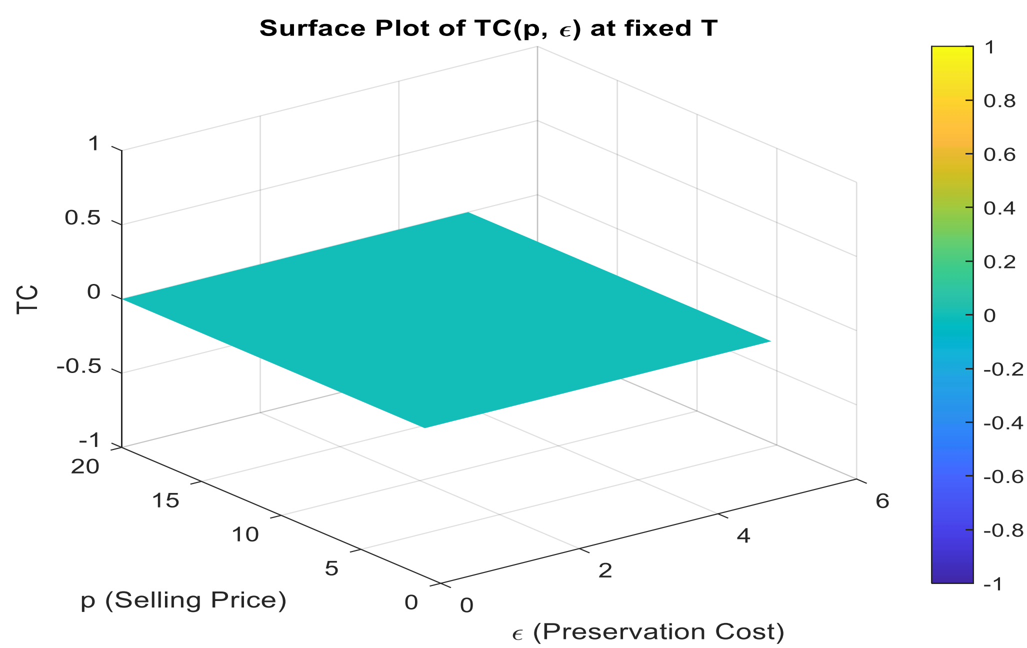

Fig:2 graph between selling price and preservation technology

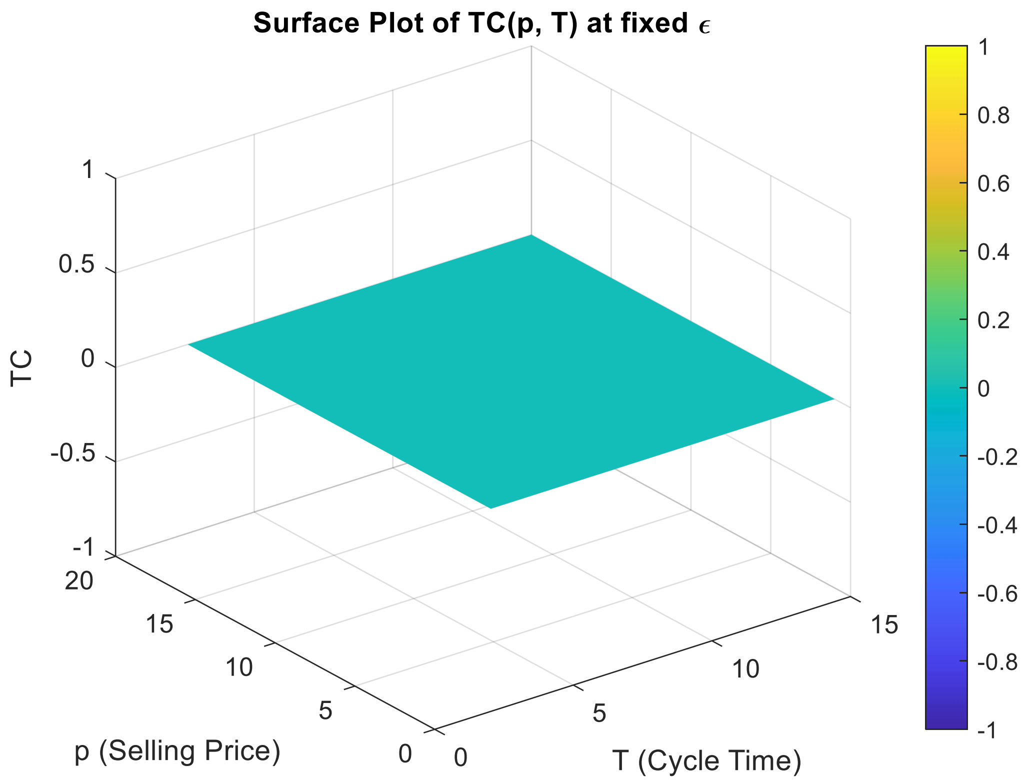

Fig:3- graph between selling price and cycle time

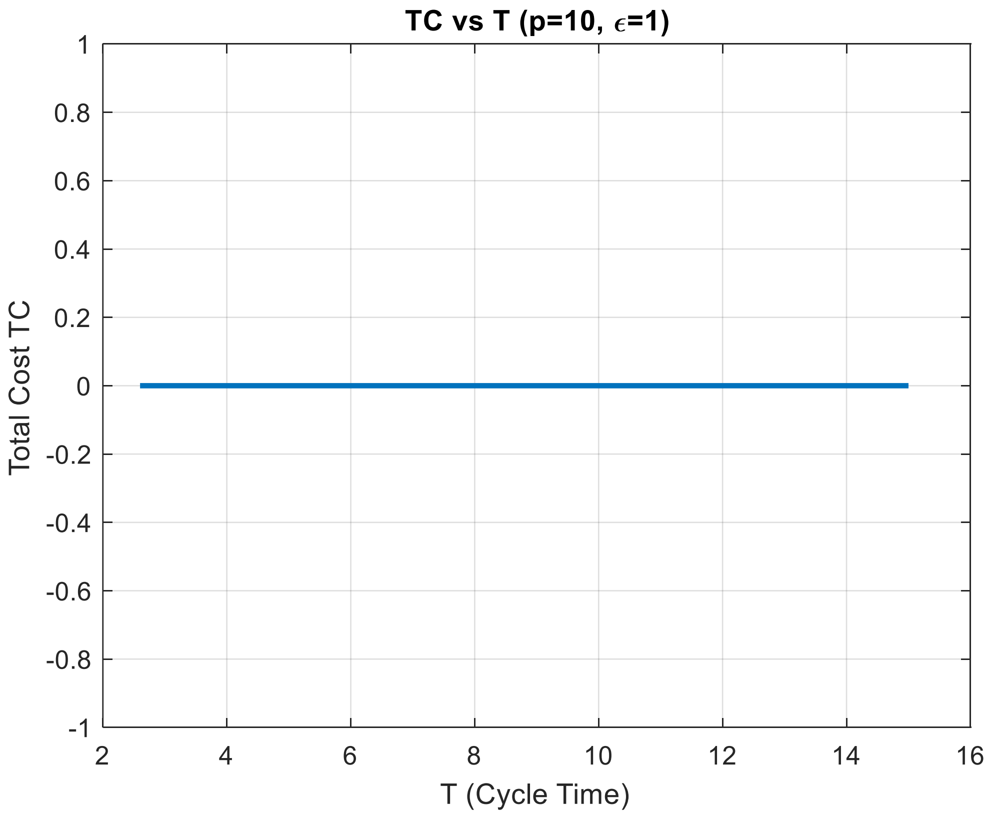

Fig:4- graph between selling total cost and cycle time.

Now in next section we show the sensitivity analysis for different parameters.

6. Sensitivity analysis

The sensitivity analysis was conducted by varying each cost parameter by +20%, +10%, 0%, −10%, and −20% while keeping the baseline optimal solution constant. The observations are summarized below.

Parameter	+20%	+10%	0%	-10%	-20%
O	101.0	100.5	100.0	99.5	99.0
Cp	105.0	102.5	100.0	97.5	95.0
Ch	112.0	106.0	100.0	94.0	88.0
β	90.0	95.0	100.0	105.0	110.0

r	96.0	98.0	100.0	102.0	104.0
Sc	106.0	103.0	100.0	97.0	94.0
Cl	103.0	101.5	100.0	98.5	97.0
γ1	102.0	101.0	100.0	99.0	98.0
γ2	101.5	100.75	100.0	99.25	98.5

Parameter	+20%	+10%	0%	-10%	-20%
O_cost	8.8157	8.7720	8.7284	8.6848	8.6411
Cp	9.1649	8.9437	8.7284	8.5082	8.2910
Ch	9.776	9.252	8.7284	8.2137	7.682
β	7.8556	8.2920	8.7284	9.1649	9.6013
r	8.3793	8.5530	8.7284	8.9029	9.0783
Sc	9.2521	8.9902	8.7284	8.4665	8.2046
Cl	8.9902	8.8593	8.7284	8.5975	8.4665
γ1	8.9029	8.8157	8.7284	8.6411	8.5530
γ2	8.8593	8.7990	8.7284	8.6578	8.5950

7. Observations

- Holding cost (Ch) is the most dominant factor influencing total inventory cost.
- Purchasing cost (Cp) has noticeable but moderate effect.
- Demand exponent (β) has a unique reverse influence, significantly shaping cost behavior through demand elasticity.
- Discount rate (r) affects cost mildly but consistently.
- Shortage and lost sale costs have moderate effects and should be controlled for optimal performance.
- Carbon emission costs (γ_1, γ_2) have the least impact.
- Ordering cost is almost negligible in its effect on the total cost.

8. Managerial insights

- Reducing holding cost yields the greatest total cost savings.
- Pricing strategy should integrate the demand exponent β due to its strong inverse effect on cost.
- Shortage and lost sale costs should be monitored, but they are not the primary drivers.
- Carbon related costs, although necessary for sustainability, have minimal economic impact.
- Ordering cost is not a key factor in cost optimization

9. Conclusion

This study develops a comprehensive inventory model that incorporates several realistic features, including price-sensitive demand, preservation technology investment, carbon emissions, inflationary effects, and time-dependent holding costs. By integrating environmental sustainability factors—specifically carbon emissions during holding and deterioration—alongside economic considerations, the model provides a more practical framework for modern inventory management. The joint optimization of selling price, replenishment cycle length, and preservation technology investment allows decision-makers to balance profitability with environmental responsibility.

Using MATLAB software, the model was computationally implemented and solved efficiently, enabling precise numerical analysis and scenario evaluations. MATLAB's optimization capabilities helped validate the proposed framework and illustrated how optimal decisions shift under varying inflation rates, sustainability constraints, and preservation investment levels. The results offer valuable managerial insights, demonstrating that incorporating preservation technology and emission considerations can significantly enhance both economic and environmental performance. Overall, the study underscores the importance of integrating sustainability into inventory decision-making and highlights MATLAB as an effective tool for analyzing complex, inflation-affected inventory systems.

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Code

```
% TC_minimization_safe.m

clc; clear; close all


%% ---------------- PARAMETERS (edit these sensibly) ----------------

O_cost = 30;

Cp = 10;

eta = 80;

beta = 1.2;

alpha = 0.15;      % lambda(eps) = alpha * eps

Ch = 2;
```

```
Cd = 1.5;
Sc = 4;
Cl = 3;
gamma1 = 0.5;
gamma2 = 0.3;
T1 = 1.0;
T2 = 2.5;
r = 0.04;
delta_param = 0.2;

params = struct('O_cost',O_cost,'Cp',Cp,'eta',eta,'beta',beta,'alpha',alpha, ...
    'Ch',Ch,'Cd',Cd,'Sc',Sc,'Cl',Cl,'gamma1',gamma1,'gamma2',gamma2, ...
    'T1',T1,'T2',T2,'r',r,'delta_param',delta_param);

% lambda(eps) definition
params.lambda_fun = @(eps) params.alpha * eps;

%% ------------- Objective as function handle -------------
obj = @(x) TC_eval_safe(x(1), x(2), x(3), params);

%% ------------- Optimization settings --------------------
% Stronger, realistic bounds:
% p (price) min  = 1, max = 100
% eps (preservation cost) min = 0.01, max = 10
% T (cycle time) min = T2 + 0.1, max = 100
```

```
x0 = [10, 1, 6];   % initial guess
lb = [1, 0.01, params.T2 + 0.1];
ub = [100, 10, 100];

% Nonlinear constraint: ensure T >= T2 + small margin
nonlcon = @(x) deal(params.T2 + 0.0001 - x(3), []);

options = optimoptions('fmincon','Display','iter','Algorithm','sqp', ...
    'MaxFunctionEvaluations',5e4,'OptimalityTolerance',1e-6);

[x_opt,fval_opt,exitflag,output] = fmincon(obj,x0,[],[],[],[],lb,ub,nonlcon,options);

%% --------------- Display results ------------------------
fprintf('\nResult (safe run):\n');
fprintf('p* = %.6f\n', x_opt(1));
fprintf('eps* = %.6f\n', x_opt(2));
fprintf('T* = %.6f\n', x_opt(3));
fprintf('TC* = %.6f\n', fval_opt);
fprintf('exitflag = %d\n', exitflag);
disp(output.message);

%% --------------- Local function: TC evaluator (safe) -----------
function tc = TC_eval_safe(p, eps, T, params)
    % guard
```

```
if p <= 0 || T <= 0 || eps < 0
    tc = 1e12; return
end

% unpack
O_cost = params.O_cost; Cp = params.Cp; eta = params.eta; beta = params.beta;
lambda_fun = params.lambda_fun; Ch = params.Ch; Cd = params.Cd; Sc = params.Sc;
Cl = params.Cl; gamma1 = params.gamma1; gamma2 = params.gamma2;
T1 = params.T1; T2 = params.T2; r = params.r; delta_param = params.delta_param;

lambda_eps = lambda_fun(eps);
e_lambda_dt = exp(lambda_eps * (T2 - T1));
A = eta / (p^beta);

e_rT1 = exp(-r * T1);
e_rT2 = exp(-r * T2);
e_rT  = exp(-r * T);
e_lambda_r_T1 = exp(lambda_eps * (T2 - T1) - r * T1);

% Compose subterms but ensure positive contributions using abs()
t1_part = T1 * ( (T1 * e_rT1)/(-r) - (e_rT1 - 1)/r^2 ) ...
        - ( (T1^2 * e_rT1)/(-r) - (2*T1 * e_rT1)/r^2 - 2*(e_rT1 - 1)/r^3 ) ...
        + ( (e_lambda_dt - 1) * ( (T1 * e_rT1)/(-r) + (e_rT1 - 1)/r^2 ) );

t2_part = ( (T2 * e_rT2 - T1 * e_lambda_r_T1)/( -(lambda_eps + r) ) ...
```

```
    - (e_rT2 - e_lambda_r_T1)/((lambda_eps + r)^2) ...
    + (T2 * e_rT2 - T1 * e_rT1)/r ...
    + (e_rT2 - e_rT1)/r^2 );

t3_part = T2 * ( (T * e_rT - T2 * e_rT2)/(-r) - (e_rT - e_rT2)/r^2 ) ...
    - ( (T^2 * e_rT - T2^2 * e_rT2)/(-r) - 2*(T * e_rT - T2 * e_rT2)/r^2 - 2*(e_rT - e_rT2)/r^3 );

% Use absolute to ensure costs add positively (robust fix)
sum_hold_integrals = abs(t1_part) + abs(t2_part) + abs(t3_part);

term_O = O_cost;
term_Cp = Cp * ( A * T1 + A * (e_lambda_dt - 1) );

term_Ch = Ch * ( A * sum_hold_integrals );
term_Cd = Cd * abs(lambda_eps) * A * abs(t2_part);

shortage_piece = T2 * (e_rT - e_rT2)/(-r) - ( (T * e_rT - T2 * e_rT2)/(-r) - (e_rT - e_rT2)/r^2 );
% Make shortage cost positive
term_short = + eps * Sc * A * abs(shortage_piece);

term_Cl = Cl * (1 - delta_param) * A * abs( (e_rT - e_rT2)/(-r) );
term_gamma1 = gamma1 * ( A * sum_hold_integrals );
term_gamma2 = gamma2 * ( abs(lambda_eps) * A * abs(t2_part) );
```

```
    inside = term_O + term_Cp + term_Ch + term_Cd + term_short + term_Cl + term_gamma1 + term_gamma2;

    tc_val = (1 / T) * inside;

    if ~isfinite(tc_val) || ~isreal(tc_val)
        tc = 1e12;
    else
        tc = double(real(tc_val));
    end
end
```

Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
0	4	1.155563e+02	0.000e+00	1.000e+00	0.000e+00	4.882e+01
1	8	1.887453e+01	0.000e+00	1.000e+00	1.373e+01	6.509e+01
2	31	1.881838e+01	0.000e+00	1.140e-03	3.151e-02	2.822e+01
3	38	1.728512e+01	0.000e+00	3.430e-01	4.194e+00	3.834e+01
4	42	1.591632e+01	0.000e+00	1.000e+00	6.258e-01	1.326e+01
5	46	1.545340e+01	0.000e+00	1.000e+00	6.616e-01	1.012e+01
6	50	1.514514e+01	0.000e+00	1.000e+00	9.399e-01	9.373e+00
7	54	1.386467e+01	0.000e+00	1.000e+00	5.379e+00	1.050e+01
8	58	1.219580e+01	0.000e+00	1.000e+00	1.086e+01	7.264e+00
9	62	1.070894e+01	0.000e+00	1.000e+00	1.631e+01	4.845e+00
10	66	9.470406e+00	0.000e+00	1.000e+00	2.299e+01	3.248e+00
11	70	8.869655e+00	0.000e+00	1.000e+00	1.640e+01	2.624e+00

12	74	8.863086e+00	0.000e+00	1.000e+00	3.585e-02	2.623e+00
13	78	8.846917e+00	0.000e+00	1.000e+00	1.501e-01	2.621e+00
14	82	8.844764e+00	0.000e+00	1.000e+00	4.491e-02	2.621e+00
15	86	8.842172e+00	0.000e+00	1.000e+00	2.927e-02	2.620e+00
16	90	8.830911e+00	0.000e+00	1.000e+00	8.002e-02	2.619e+00
17	94	8.805650e+00	0.000e+00	1.000e+00	1.086e-01	2.616e+00
18	98	8.778601e+00	0.000e+00	1.000e+00	5.092e-02	1.119e+00
19	102	8.749032e+00	0.000e+00	1.000e+00	1.487e-01	2.605e+00
20	106	8.732113e+00	0.000e+00	1.000e+00	1.811e-01	8.843e-01
21	110	8.730271e+00	0.000e+00	1.000e+00	1.965e-02	8.986e-01
22	119	8.729658e+00	0.000e+00	1.681e-01	1.144e-02	2.601e+00
23	124	8.729196e+00	0.000e+00	7.000e-01	6.299e-03	9.026e-01
24	128	8.728896e+00	0.000e+00	1.000e+00	8.956e-04	9.021e-01
25	135	8.728612e+00	0.000e+00	3.430e-01	6.316e-04	2.601e+00
26	141	8.728565e+00	0.000e+00	4.900e-01	3.339e-04	9.019e-01
27	145	8.728517e+00	0.000e+00	1.000e+00	2.893e-04	9.017e-01
28	152	8.728450e+00	0.000e+00	3.430e-01	1.688e-04	2.601e+00
29	160	8.728448e+00	0.000e+00	2.401e-01	1.691e-05	9.016e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
30	164	8.728444e+00	0.000e+00	1.000e+00	4.594e-06	9.016e-01
31	171	8.728438e+00	0.000e+00	3.430e-01	7.975e-06	9.016e-01
32	185	8.728438e+00	0.000e+00	6.782e-03	7.241e-07	9.016e-01

Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than

the value of the step size tolerance and constraints are

satisfied to within the value of the constraint tolerance.

<stopping criteria details>

Result (safe run):

p* = 100.000000

eps* = 0.014938

T* = 4.981316

TC* = 8.728438

exitflag = 2

Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than

the value of the step size tolerance and constraints are

satisfied to within the value of the constraint tolerance.

<stopping criteria details>

Optimization stopped because the relative changes in all elements of x are

less than options.StepTolerance = 1.000000e-06, and the relative maximum constraint

violation, 0.000000e+00, is less than options.ConstraintTolerance = 1.000000e-06.

>>